Variable Selection in Poisson Regression Model using Golden Jackal Optimization Algorithm

Didar Abdal Wafaa Rashid Aziz[1], Mohammad Mahmood Faqe Hussein[2], Zakariya Yahya Algamal[3]

[1] College of Administration and Economics, University of Sulaimani- Sulaimani city – Iraq, didar.rashid@univsul.edu.iq

[2] Statistics and informatics department - College of Administration and Economics -University of Sulaimani- Sulaimani city – Iraq, Mohammad.faqe@univsul.edu.iq

[3] College of Computer Sciences and Mathematics, University of Mosul, zakariya.algamal@uomosul.edu.iq

Abstract. The Poisson regression model is one of the most important logarithmic linear regression models, and it is the tool through which the response variable is modeled when the values of that variable are in the form of countable values. Like other regression models, the model may contain many explantory variables, which negatively affects the accuracy of the model and its simplicity in interpreting the results. This study aims to use the Golden Jackal algorithm and compare it with other methods in selecting variables in the Poisson regression model using simulation and real data. The Monte-Carlo method was used in the simulation to generate data that follow the Poisson regression model according to different factors such as sample size and the number of explantory variables. Two aspects of the performance evaluation of the methods used were relied upon: the first is to evaluate the accuracy of prediction and the second is to evaluate the selection of variables as a criterion for comparison. The simulation results showed the superiority of the Golden Jackal algorithm compared to other variable selection methods. In addition, the application was carried out on real data collected from patients with chronic kidney disease who are treated with continuous hemodialysis, and the patients' condition was diagnosed by specialist doctors in cooperation with Ibn Sina Teaching Hospital - Artificial Kidney Unit.

Keywords: Golden Jackal algorithm, Poisson regression, Variable selection, Simulation experiment description

1. INTRODUCTION

Regression analysis is a statistical tool that builds a statistical model to estimate the relationship between one variable called the dependent variable and another variable or several other variables called explanatory variables, so that a statistical equation is produced that explains the relationship between the variables. Regression analysis with its various models has occupied a distinguished position in the trends of many statisticians, and has received its abundant share through various statistical writings, and its role has become very important in the applications of various life sciences, especially in the economic field, which has taken it upon itself to adopt regression models primarily

to be the most prominent means of practical support for economic theories, in addition to other sciences such as health, life, social, and others (Al-Rawi, 1978).

The classical linear regression model assumes that the response variable depends on a set of explanatory variables, where these variables can be continuous variables or countable variables. However, when the response variable is in the form of countable variables such as the number of patients, the assumptions of linear regression will not be achieved. Therefore, the Poisson regression model was proposed as one of the regression models that are compatible with such cases.

Selecting variables in count data using the Poisson regression model is one of the challenges in applying the Poisson regression model when the number of explanatory variables is large, as traditional methods for selecting subsets such as forward selection, backward elimination, and stepwise selection have become poor in performing their function as they have become more expensive to calculate. In addition, information criteria for selecting variables such as the Akaike information criterion (AIC) and the Bayesian information criterion (BIC) have become impractical in selecting explanatory variables due to their computational complexity, which grows exponentially with the increase in the number of explanatory variables (Algamal, 2015).

The current study dealt with the Poisson Regression Model (PRM), which is one of the most popular models among models that have a countable response variable. It was first described by researchers Nelder and Wedderburn (1972), as a special case of generalized linear models (GLMs). To determine the importance of the methodology compared to other traditional methods, the model used will be subjected to and then the criteria for evaluating the significance of the results of each method will be employed. This research aims to employ the Golden Jackal algorithm and compare it with other methods for selecting explanatory variables in the Poisson regression model using simulation and real data, by highlighting a number of factors that may affect the quality of these methods and the necessity of using them under certain conditions rather than other methods.

2. POISSON REGRESSION MODEL

The Poisson regression model is one of the most important logarithmic linear regression models, and it is the tool through which the dependent variable is modeled when the values of that variable are in the form of countable values. Like other regression models, the model may contain many independent variables, which negatively affects the accuracy of the model and its simplicity in interpreting the results. This model assumes that the dependent variable y_i is a response variable that

follows the Poisson distribution with a parameter of μ, and the random errors in the model follow the Poisson distribution with a parameter of μ. (Hossain And Ahmed, (2012)) Mansson and Kubria (2012) and is defined according to the probability function defined by the following formula.

$$y_i = e^{X\beta + U} \qquad \ldots (1)$$

It can also be expressed in matrix form as follows:

$$y_i = Exp(X\beta + U) \qquad \ldots (2)$$

Since:

y_i: Dependent variable vector ,

$X_{n \times 1}$: Independent variables matrix with degree $\beta_{(n \times (p+1))}$

β: Parameter vector with degree $U_{((p+1) \times 1)}$

U: Random errors vector with degree $n_{(n \times 1)}$

n: Sample size

P : Number of independent variables (explanatory).

In order to estimate the parameters of the Poisson regression model using the probability methods, we will resort to maximizing the observations of the distribution of the dependent variable y_i . If the dependent variable y_i follows the Poisson distribution with a parameter of (μ_i) then the distribution function is as in formula (1) and is defined in advance as follows:

$$f(y_i/\mu_i) = \frac{e^{-\mu i} \, \mu_i^{y_i}}{y_i!}$$

By maximizing the observations of the distribution of the dependent variable y_i given in the formula above, the maximum likelihood function is as follows:

$$L(y_1, y_2, \ldots, y_n; \mu_i) = \frac{Exp\{-\sum_{i=1}^n \mu_i\} \, \mu_i^{\sum_{i=1}^n y_i}}{\prod_{i=1}^n y_i!} \qquad \ldots (3)$$

Taking the natural logarithm of the maximum likelihood function for the above observations, we get:

$$LogL(y_i|x_i, \beta) = -\sum_{i=1}^n \mu_i + \sum_{i=1}^n y_i (Log\{\mu_i\}) - Log\left\{\prod_{i=1}^n y_i!\right\} \quad \ldots (4)$$

Based on the second assumption of the basic assumptions of the Poisson regression model $\mu_i = Exp\{X_i^T\beta\}$, this assumption is replaced by function (4) above as follows:

$$LogL(\mathbf{y_i}|x_i, \boldsymbol{\beta}) = -\sum_{i=1}^n \left(Exp\{x_i^T\boldsymbol{\beta}\}\right) + \sum_{i=1}^n y_i \left(Log\left\{Exp\{x_i^T\boldsymbol{\beta}\}\right\}\right) - Log\left\{\prod_{i=1}^n y_i!\right\}$$

$$= \sum_{i=1}^{n} \left(y_i x_i^T \boldsymbol{\beta} - \text{EXP}(x_i^T \boldsymbol{\beta}) - \log y_i! \right) \qquad \dots (5)$$

3. 3 GOLDEN JACKAL OPTIMISATION ALGORITHME

Most of the optimization algorithms inspired by nature are based on swarm intelligence, and swarm intelligence-based algorithms constitute a large part of contemporary algorithms, and these algorithms have become widely used in classification, optimization, image processing, business intelligence, as well as in machine learning and artificial intelligence. The Golden Jackal Optimization Algorithm (GJOA) is one of the latest swarm intelligence methods and the most powerful optimization algorithms that was first developed in 2022 by Chopra and Ansari (Chopra & Mohsin Ansari, 2022).

The golden jackal algorithm has been proven to be effective and perform well in solving various optimization problems. The golden jackal algorithm was developed by simulating its hunting behavior. The golden jackal is a medium-sized terrestrial predator that lives in North and East Africa, the Middle East, Europe, Southeast Asia, and Central Asia. The golden jackal has a body length of about 70 to 85 cm, a standing height of about 40 cm, and a tail length of about 25 cm. The fur is usually coarse with pale golden brown to yellow tips and varies with region and season. The small body and long legs allow the golden jackal to run long distances to catch prey. The initial stages of hunting a golden jackal pair are as follows:

1- Search and advance towards the prey

2- Encircle the prey and disturb it until it stops moving

3- Pounce on the prey

The hunting strategy of the golden jackal pair is mathematically designed and the mathematical model is improved. The algorithm shows the sub-section of the golden jackal algorithm development process and to improve the algorithm as a simple and homogeneous method, the search space is formulated like many other algorithms.

The golden jackal algorithm starts by generating the initial solution uniformly over the search space as follows:

$$Y_0 = Y_{min} + \text{rand}(Y_{max} - Y_{min}) \qquad \dots\dots\dots\dots (٦)$$

Where:

y_max: The upper limit of the variables.

y_min: The lower limit of the variables, which is a random variable that follows the regular position within the interval 0 and 1. Thus, we obtain the initial matrix of the search space values, which is as follows:

$$\text{Prey} = \begin{bmatrix} Y_{1,1} & Y_{1,2} & \cdots & Y_{1,d} \\ Y_{2,1} & Y_{2,1} & \cdots & Y_{2,d} \\ \vdots & \vdots & \vdots & \vdots \\ Y_{n,1} & Y_{n,2} & \cdots & Y_{n,d} \end{bmatrix} \quad \ldots\ldots\ldots\ldots (7)$$

Where:

y_{ij}: refers to the dimension j of the prey i.

n: total of prey and variables.

d: refers to the position of the prey for the parameters.

Then the fitness function (objective function) is applied to each row of the matrix that represents the prey shown in equation (7) as follows:

$$F_{OA} = \begin{bmatrix} f(Y_{1,1}; Y_{1,2}; \cdots; Y_{1,d}) \\ f(Y_{2,1}; Y_{2,1}; \cdots; Y_{2,d}) \\ \vdots \\ f(Y_{n,1}; Y_{n,2}; \cdots; Y_{n,d}) \end{bmatrix} \quad \ldots\ldots\ldots\ldots (8)$$

Where F_{OA} is the matrix of fitness function values for each prey and y_{ij} shows the value of dimension j for prey i.

n: number of prey.

The algorithm method depends on two stages: the exploration stage and the exploitation stage.

First: The exploration or search phase for prey

A strategy was proposed to explore the golden jackal algorithm. The nature of the jackal is to know how to perceive the prey and follow it intermittently. The prey cannot be caught easily, so the jackal waits and searches for another prey. Note that the hunting is led by the male jackal and followed by the female jackal.

$$Y_1(t) = Y_M(t) - E \cdot |Y_M(t) - rl \cdot \text{Prey}(t)| \qquad \ldots\ldots\ldots (9)$$

$$Y_2(t) = Y_{FM}(t) - E \cdot |Y_{FM}(t) - rl \cdot \text{Prey}(t)| \qquad \ldots\ldots\ldots (10)$$

Where:

16

t : Current iteration.

Prey (t): Position vector of the prey.

$Y_M(t), Y_{FM}(t)$: Indicates the position of the male and female jackal.

$Y_1(t), Y_2(t)$:The prey corresponds to the male and female jackal positions updated by the expression $Y_1(t), Y_2(t)$

E: Evasion energy of the prey and is calculated as follows:

$$E = E_1 \times E_0 \qquad \dots\dots\dots\dots(11)$$

Where:

E_1: refers to the decreasing energy of the prey.

E_0: refers to the initial state of its energy

$$E_0 = 2 \times r - 1 \qquad \dots\dots\dots\dots(12)$$

Where r is a random number between.$(0,1)$

$$E_1 = c_1 \times \left(1 - \left(\frac{t}{T}\right)\right) \qquad \dots\dots\dots\dots\dots(13)$$

(T) denotes the maximum number of iterations and C1 denotes a constant value equal to 1.5. E_1 is linearly reduced from 1.5 to 0. $|y(t) - r| \cdot \text{Prey}(t)$ in Equation (11) and (12) calculates the distance between the jackal and the prey. This distance between the jackal and the prey is either subtracted or added. The current position of the jackal depends on evading the energy of the prey. r in Equation (11) is a parabola and in Equation (12) is a vector of random numbers based on the (Légène) distribution representing the motion of a doubled Légène r and Prey the motion of the prey. It is calculated using

$$r_1 = 0.05 \times LF(y) \qquad \dots\dots\dots\dots(14)$$

It is a tax flight function which is calculated using

$$LF(y) = 0.01 \times \frac{\mu \times \sigma}{\left(\left|v^{\left(\frac{1}{\beta}\right)}\right|\right)}; \sigma = \left(\frac{\Gamma(1+\beta) \times \sin\left(\frac{\pi\beta}{2}\right)}{\Gamma\left(\frac{1+\beta}{2}\right) \times \beta \times \left(2^{\frac{1-1}{2}}\right)}\right)^{\frac{1}{\beta}} \qquad \dots\dots(15)$$

Where:

μ and v: random values between (0 and 1).

β: is a virtual constant set to 1.5.

The golden jackal algorithm is updated by taking the average of equation (4) and equivalent.(٥)

$$Y(t + ١) = \frac{Y_١(t) + Y_٢(t)}{٢} \qquad \dots\dots\dots\dots (١٦)$$

The exploitation stage or encirclement and attack on the prey when the prey is harassed by the jackal

$$Y_١(t) = Y_M(t) - E.|r_1.Y_M(t) - Prey(t)| \qquad \dots\dots\dots (١٧)$$

$$Y_٢(t) = Y_{FM}(t) - E.|r_1.Y_{FM}(t) - Prey(t)| \qquad \dots\dots (١٨)$$

The evasion energy of the prey E is calculated according to equation (4) and finally the equivalence of the hyena is updated according to equation (16) and the r_1 function in equation (17) and the equivalent (18) is the arbitrary progress of behavior in the exploitation phase and the preference for exploration and local avoidance is optimal and r_1 is calculated according to equation (14) This element helps in avoiding local slowdown especially in the conclusion.

<u>**Second:** **Transition from exploration to exploitation**</u>

In the golden jackal algorithm, the energy escaping from the prey is used to transition from exploration to exploitation. The energy of the prey decreases significantly during evasion behavior. Considering this prey, the energy evasion model is designed according to the equation The initial energy E_0 arbitrarily deviates from (-1 to 1) in each iteration When E_0 decreases from (0 -1) the prey is physically diminished Although the value of E_0 increases from (0 to 1) it indicates an improvement in the prey's strength The variable evasion strength E_0 decreases during the iteration process When E >1, the jackal pairs search in different sections to explore the prey and when E <1, the golden jackal attacks the prey and exploitation finally rescues it. The search process in the golden jackal algorithm begins by generating a random population of prey. During iterations, the potential location of the prey is estimated by hunting a male and female jackal pair. Each candidate in the population updates its distance from the jackal pair. The parameter E_1 is reduced from (1.5 to 0) to ensure exploration and exploitation, respectively. The golden jackal hunting pair deviates from the prey when E >1 and clusters with the prey when E <1. The golden jackal algorithm is completed by meeting the termination criterion by returning the pseudocode of the golden jackal algorithm. Figure 1 illustrates how the algorithm works in selecting variables.

x_1	x_2	··········	x_{p-1}	x_p
1	0	··········	1	0

Fig. 1: Variable selection mechanism according to the Golden Aviary algorithm.

Third: **Criteria for evaluating penalty methods**

The method of evaluating the performance of penalty methods, comparing these methods among themselves, and choosing the best method is an important aspect of data analysis. In general, there are two aspects of evaluating the performance of penalty methods: the first is evaluating the accuracy of prediction and the second is evaluating the choice of variables

Fourth: Criteria for evaluating the accuracy of prediction

First: Estimation Error (EE)

It is defined as the square of the difference between the value of the actual parameters and the value of the estimated parameters and is defined in the following mathematical form

$$:EE = \left(\hat{\beta} - \beta\right)^{T}\left(\hat{\beta} - \beta\right) \qquad \dots(9)$$

Where: $:\hat{\beta}$ is the vector of parameters estimated according to the methods used and β is the vector of real parameters.

Second: Prediction Error (PE)

It is defined as the square of the difference between the true value of the response variable and its associated predictive value, and is defined mathematically by the following equation:

$$PE = (y - \hat{y})^{T}.(y - \hat{y}) \qquad \dots(10)$$

Where $\hat{y} = Exp\{X^{T}\beta\}$ and based on these two criteria the best method is determined which gives the lowest value compared to the other methods.

Fifth: Criteria for evaluating the accuracy of variable selection

Since the proposed methods in general (I, C) work on selecting variables, it is important to evaluate and measure the ability of these methods and their quality in how to select important variables. Therefore, two criteria were relied upon in our study for this purpose as follows:

First: Evaluation Criterion "C"

It is the evaluation criterion symbolized by (C) which is defined as the number of real transactions with zero values that were correctly estimated as having zero values.

Second: Evaluation Criterion "I"

The evaluation criterion symbolized by (I) which is defined as the number of real transactions with non-zero values that were incorrectly estimated as having zero values. The quality of penalty methods in terms of criteria for evaluating the accuracy of variable selection depends on who gives the highest value for (C) and the lowest value for (I).

4. DESCRIPTION OF THE SIMULATION EXPERIMENT

The experiment was designed and simulated using the programming language (R) where the variable (y_i) was generated in the Poisson regression model that follows the Poisson distribution with a rate of (μ_i), where the Monte Carlo method was used in the simulation where the values of the sample size (n) were set where three sample sizes were used which are (50, 100, 150) in order to study the comparison according to the samples of different types. The comparison will be made with both the LASSO method which represents the abbreviation of Least Absolute Shrinkage and Selection Operator as well as the SCAD method which represents the abbreviation of Smoothly Clipped Absolute Deviation.

5. SIMULATION STUDIES

First: The data for the variable y were generated, which follow the Poisson regression model as follows:

$$y \sim P(\exp(X\beta))$$

Second: The matrix of explanatory variables X was generated with dimensions (n × p) that follow the multivariate normal distribution (Multivariate Normal Distribution) as follows:

$$X \sim MN(\mu, M)$$

Where M is the covariance matrix, where $M_{ij} = r^{|i-j|}$, when (i,j=1,2,…,p) where the explanatory variables are correlated.

Third: The experiment was repeated (100) times in order to reduce bias in Monte Carlo experiments .

Fourth: The data of the Poisson regression model were generated according to the values of the regression parameter vector β, whose dimensions are ($1 \times p$). The values of the regression parameter vector β were as follows: $\beta = (0, …, 0, 1.5, -0.4, 0.8, -0.6, 1.2)^T$, where the number of non-zero parameters is q=5, and the zero parameters are equal to p-q.

6. INTERPRETATION OF SIMULATION RESULTS

The results of the simulation experiment will be analyzed and interpreted according to the criteria of prediction accuracy and the criteria of variable selection accuracy. By observing Table (1), (2) and (3) which shows the values of the criteria of each of (EE, PE, C, I) for the penalty methods (LASSO, SCAD) proposed by researchers Zou (2006), Zou and Hastie (2005), Tibshirani (1996), Fan and Li (2001), El-Anbari and Mkhadri (2013) and the proposed method GJO, the following can be concluded:

1- When the correlation coefficient between the variables changes from (0.5) to 0.7, it is clear that the (GJO) method gave the lowest values of (EE, PE) as the amount of improvement in prediction based on the PE criterion reached 69.36% and 2.3% at (r=0.5) and 63.63% and 2.83% at (r=0.7) compared to LASSO) and (SCAD respectively, and the improvement in the estimation error based on the criterion (EE) reached By 99.01% and 45.22% at (r=0.5) and 97.66% and 39.34% at (r=0.7) compared to (LASSO and SCAD) respectively.

2- When the correlation coefficient is equal to (0.9), the (GJO) method gave the best results compared to other methods, as the prediction improved based on the PE criterion by 52.59% and 7.28% compared to (LASSO and SCAD) respectively, and the improvement in the estimation error based on the EE criterion reached 94.90% and 78.77% compared to (LASSO and SCAD) respectively.

3- Depending on the criteria for selecting variables, the (GJO) method had the highest values of (C)) which is the number of true coefficients with zero values that were correctly estimated as having zero values, and gave the lowest values of (I) which is known as the number of true coefficients with non-zero values that were incorrectly estimated as having zero values at the values of the correlation coefficient (0.5) and 0.7). While the (GJO) method showed a discrepancy in the criteria for selecting variables at the value of the correlation coefficient (0.9).

4- The (LASSO) method appeared as the worst method in estimation because it gives the highest values for (PE and EE) and also as the worst method in selecting variables because it tends to select unimportant explanatory variables.

Table (1): Average criteria for evaluating penalty methods when P=10 and n=50

r	Method	PE	EE	C	I
0.5	LASSO	32.3507	2.1488	1	0
	SCAD	10.1541	0.0387	4	0
	GJO	9.8014	0.0281	5	0
0.7	LASSO	29.9037	2.0302	3.5	0
	SCAD	10.7742	0.0783	4	0

	GJO	10.1074	0.0463	5	0
0.9	LASSO	24.1644	1.9384	4	1
	SCAD	12.3546	0.4654	4.5	0
	GJO	11.328	0.2416	5	0

Table (2): Average criteria for evaluating penalty methods when P=10 and n=100

r	Method	PE	EE	C	I
0.5	LASSO	19.3353	2.0341	2	0
	SCAD	6.9986	0.0699	4	0
	GJO	6.6514	0.0432	5	0
0.7	LASSO	18.6220	1.8992	3	0
	SCAD	7.5870	0.1520	4	0
	GJO	6.8547	0.0890	5	0
0.9	LASSO	14.8017	1.7818	5	1
	SCAD	8.5148	0.6986	5	1
	GJO	7.9514	0.5511	6	1

Table (3): Average criteria for evaluating penalty methods when P=10 and n=150

r	Method	PE	EE	C	I
0.5	LASSO	8.4941	1.7540	4	0
	SCAD	3.8819	0.1872	4	0
	GJO	3.1247	0.1288	4	0
0.7	LASSO	7.7433	1.6771	4	0
	SCAD	4.2797	0.4467	4	0
	GJO	3.9825	0.3064	5	0
0.9	LASSO	6.2328	1.8516	5	2
	SCAD	4.7625	1.1938	5	2
	GJO	4.2961	1.3040	5	1

7. PRACTICAL PART

In order to complete the desired benefit of the research, the application was made to the Poisson distribution tracking data which was taken from data used by (Liqa Saeed et al., 2011) on chronic renal failure disease, where (73) blood samples were collected from people with chronic renal failure disease who are treated with continuous hemodialysis, and blood samples were drawn from the group of patients before the hemodialysis process which takes (3-4) hours, and the patients' condition was diagnosed by specialist doctors in cooperation with Ibn Sina Teaching Hospital - Artificial Kidney Unit, their ages ranged between (20-80) years, and included (38) male models and (35) female models, and the patients' information was recorded according to a special questionnaire form for each patient prepared for this purpose for the year 2013, where the study recorded eight explanatory

variables which are believed to have an effect on the response variable which represents the number of times of hemodialysis per month. Table (4) shows a description of the explanatory variables used in the study.

Table (4): Description of the independent variables used in the study

Unit of measure	Explanatory variable description	Explanatory variable symbol
(Male = 1, Female = 2)	Gender	X1
Years	Age	X2
Days	Duration of disease	X3
(Yes = 1, No = 2)	Genetics	X4
(mmol/L)	Urea ratio	X5
g/100ml	Total protein ratio	X6
g/100ml	Albumin ratio	X7
g/100ml	Globulin ratio	X8

The parameters of the Poisson regression model are estimated by the maximum likelihood estimator (MLE) regardless of the estimate of ($(\beta_0$, then the values of $(\hat{Y})$ are found to calculate the mean square error (MSE) of the model. By observing Table (5), which shows the results of the mean square error of the estimated model that were obtained, we notice that the (GJO) method is superior to the other estimation methods used, as it gave the lowest value for the mean square error, which makes it the best estimation method, then the (SCAD) method comes in second place in terms of the value of the mean square error, and the (MLE, LASSO) methods were the worst two methods as they gave the highest values for the mean square error.

Table (5): Results of the methods used based on the MSE criterion In the data of patients with renal failure

Methods	MSE
MLE	9.358487
LASSO	7.877187
GJO	5.1036
SCAD	6.9741

8. CONCLUSIONS

The simulation and practical application results showed that the GJO method is better than other variable selection methods and outperformed them when the correlation between the variables is (0.5) and (0.7), as the GJO method had the lowest values of criteria (EE, PE, I) and the highest values of (C)) for all simulation models when the correlation coefficient between the variables was (0.5) and (0.7). The simulation and practical application results also showed that the LASSO method is the worst method, as the LASSO method gave the highest values of criteria (EE, PE, I) and the lowest values of (C)) for all models when the correlation coefficient between the variables was (0.5) and (0.7) and (0.9).

7 References .

[1] Al-Rawi, Khashea Mahmoud, (1978), "Introduction to Regression Analysis", University of Mosul.

[2] Sabry, Hussam Muwaffaq (2013), "Comparing Methods of Estimating Parameters of Poisson Regression Model in the Presence of Multicollinearity Problem with Practical Application", Unpublished PhD Thesis, College of Administration and Economics, University of Baghdad.

[3] Abdullah, Liqa Saeed and Aloush, Dhikra Ali and Al-Jarrah, Israa Abdul Haq, (2011), "Study of Metalloendopeptidase Enzyme and Its Relationship to Chronic Renal Failure Disease", Journal of Rafidain Sciences, Volume (22), Issue (4), pp.(٨٧-٧١) .

[4] Mehrabian, A. R., & Lucas, C. (2006). A novel numerical optimization algorithm inspired from weed colonization. Ecological informatics, 1(4), 355-366.

[5] Algamal, Z. Y. and Lee, M. H. (2015). "Penalized Poisson Regression Model using adaptive modified Elastic Net Penalty". Electronic Journal of Applied Statistical Analysis,Vol. 08, Issue 02, 236-245.

[6] El Anbari, M. and Mkhadri, A. (2013)." The adaptive gril estimator with a diverging number of parameters. Communications in Statistics - Theory and Methods". 42(14), 2634–2660.

[7] Fan, J., & Li, R. (2001). "Variable selection via nonconcave penalized likelihood and its oracle properties". Journal of the American Statistical Association, 96(456), 1348-1360.

[8] Hossain, S. and Ahmed, E. (2012). "Shrinkage and penalty estimators of a Poisson regression model". Australian & New Zealand Journal of Statistics. 54(3), 359–373.

[9] James, G., Witten, D., Hastie, T., and Tibshirani, R. (2024). "An introduction to statistical learning". Springer, New York.

[10] Månsson, K., Kibria, B. G., Sjolander, P & Shukur, G. (2012), "Improved Liu Estimators for the Poisson Regression Model", International Journal of Statistics and Probability, Vol. 1, No. 1, pp. 1-6.

[11] Tibshirani, R. (1996). "Regression shrinkage and selection via the lasso. Journal of the Royal Statistical Society". Series B (Methodological). 58(1), 267–288.

[12] Zou, H. (2006)." The adaptive lasso and its oracle properties. Journal of the American Statistical Association". 101(476), 1418–1429.

[13] Zou, H. and Hastie, T. (2005). "Regularization and variable selection via the elastic net". Journal of the Royal Statistical Society. Series B (Methodological). 67(2), 301–320.